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Preferred Frames and Oscillating Universes

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Abstract

A metric theory of gravitation is presented. It is based on the existence of a preferred 'cosmic' time. It agrees with all present experimental facts regarding gravitation and leads to singularity-free oscillating universes.

1. *Introduction*

Besides Einstein's theory of gravitation, a number of other metric theories of gravity have also been constructed for many different reasons [see Wei-Tou-Ni (1972) for a compendium of metric theories]. It appears, however, after a careful analysis by Nordtvedt & Will (1972), that theories so far built up, in which the principle of covariance has been dropped in favor of the existence of a preferred frame, are simply non-viable, they disagree 'violently' with observable facts.

To construct a viable *non-covariant* theory thus appears as a challenge.† There exists, however, a deeper reason for such an attempt: the avoidance of singularities in cosmological models. Singularities in general relativistic cosmologies, as well as in similar theories like that of Brans & Dicke (1961) in which gravitation is always attractive, seem to be inescapable; this follows from general theorems due to Penrose (1965), Geroch (1966) and Hawking (1966). In particular there seems to be no way of obtaining a real 'bounce' in an oscillating universe with a reasonable energy tensor except by abandoning general relativity and similar theories, or, perhaps [Ellis (1971)], by introducing a large repulsive ($\Lambda > 0$) cosmological term.

Various examples have been given of *covariant* theories with modified Einstein equations that lead to singularity-free cosmologies: the *C*-field theory of Hoyle & Narlikar (1964) with $\Box C = 0$, also Rosen's theory (Rosen 1969) with a cosmological field, and, lately, Trautman's version of the Einstein–

† See in this respect Wei-Tou-Ni (1973) and further remarks below.

Cartan theory with non-zero torsion related to the intrinsic spin of the universe (Trautman, 1973).

In what follows we shall briefly describe a non-covariant preferred frame theory of gravitation whose 'extended Parametrized Post-Newtonian' approximation (Will & Nordtvedt, 1972), or so-called PPN formalism, is the same as that of general relativity—irrespective of the value of a new coupling parameter† a appearing in the theory. That is, the theory agrees well with all present observational evidence. The theory also admits homogeneous and isotropic metrics that do not go through a singularity, that is, contracting and dilatating worlds, once or an infinite number of times, with read bounces.

From an esthetic point of view, abandoning the principle of covariance appears as a great loss. It may, however, be the price to pay if one is not ready to accept the idea of a closed universe reduced to one singular point with infinite density of matter. If one believes that some mechanism exists that can prevent singularities in cosmological models,‡ then 'cosmic' time should keep running. Reciprocally, by imposing the cosmic time 'to go on' we shall imply, in a self-consistent way, the existence of regular solutions. In this respect our preferred frame theory can be considered as a covariant theory with a symmetry breaking§ rather than a really non-covariant one. We shall see a new coupling constant appearing in the formalism whose non-zero value will insure a singularity-free world, but when taken infinitesimally small will lead to cosmological models as close as one wishes to those given by Einstein's equations.

2. *Lagrangian and Equations of Field with no Constraints*

Let us now proceed with formal developments. The theory is a metric one with a preferred time coordinate t, orthogonal to the three-space. The principle of covariance is thus not supposed to hold and there exists instead a preferred 'cosmic' time $-\infty < t < +\infty$, as in Rosen's non-covariant theory of gravitation (Rosen, 1971a, b). The gravitational field has seven components:||

$$ds^2 = g_{\mu\nu}\, dx^\mu\, dx^\nu \equiv \Phi^2\, dt^2 + g_{kl}\, dx^k\, dx^l \qquad (2.1)$$

in which Φ and g_{kl} are functions of t, x^k. For other fields and for matter, there exist a conserved energy tensor T^ν_μ,

$$D_\nu T^\nu_\mu = 0 \qquad (2.2)$$

and their equations, in the limit of weak gravitation (flat space) reduce to the form they have in special relativity.

Second-order differential equations of Φ and g_{kl} are derived from a

† Provided of course that the new coupling constant is not as great as to make the PPN development altogether meaningless.

‡ And there are some examples of plausible mechanisms where this happens, see for instance Parker & Fulling (1973) where other examples are cited.

§ We are indebted to G. Horwitz for having drawn our attention to this interpretation.

|| Notations: indices $\lambda, \mu, \nu = 0, 1, 2, 3$; indices $k, l = 1, 2, 3$; a covariant derivative relative to x^λ is noted D_λ, an ordinary one ∂_λ. Units: $c = 8\pi k = 1$.

Langrangian density. In accordance with our preferred frame condition we shall demand that this density L admits the following pseudo-group of coordinate transformations:

$$t' = \alpha t + \beta \qquad (\alpha, \beta \text{ real numbers}) \tag{2.3}$$

$$x'^k = f^k(x^l) \tag{2.4}$$

One can then show without difficulty that the most general polynomial homogeneous quadratic expression in time derivatives (∂_0 or a dot over a symbol) and space derivative (∂_k) of Φ and g_{kl} is, up to a divergence term, the following expression:

$$L = -[(-g)^{1/2}R + 2a\dot{u}^2] + L_M \tag{2.5}$$

in which $g \equiv \det g_{\mu\nu}$, R is the scalar curvature, $R \equiv g^{\mu\nu}R_{\mu\nu}$, L_M is the matter and other field Lagrangian density and a is an arbitrary new coupling constant while u stands for

$$u \equiv [(-g)^{1/2}g^{00}]^{1/2} \tag{2.6}$$

or with a new quantity

$$Q \equiv (-\det g_{kl})^{1/2} \tag{2.7}$$

u may be written

$$u = (Q/\Phi)^{1/2} \tag{2.8}$$

The variational derivatives of L relative to Φ and g_{kl} gives us dynamical equations. They may be written as follows:

$$G_{00} \equiv R_{00} - \tfrac{1}{2}g_{00}R - T_{00} = g_{00}a\ddot{u}/u\Phi^2 \tag{2.9}$$

$$G_{kl} \equiv R_{kl} - \tfrac{1}{2}g_{kl}R - T_{kl} = -g_{kl}a\ddot{u}/u\Phi^2 \tag{2.10}$$

They look like modified Einstein equations except that the usual 'constraint' equation† $G_{0k} = 0$ is here not an equation. In addition the correction terms are covariant for (2.3) and (2.4) and not for arbitrary coordinate transformations.

Another useful form of (2.9) and (2.10) is as follows:

$$W_{kl} \equiv G_{kl} - \tfrac{1}{2}g_{kl}G = 0 \tag{2.11}$$

$$G_{00} = a\ddot{u}/u \tag{2.12}$$

in which $G \equiv g^{\mu\nu}G_{\mu\nu}$. Equations (2.11) form the usual 'dynamical' subset of the equations of general relativity (those containing second-order time derivatives in g_{kl}). The modification introduced by our preferred frame conditions thus replaces the constraint $G_{00} = 0$ by a dynamical equation (2.12) (and suppresses the other constraint). We have here a dynamical set for Φ and g_{kl} without constraints.

† That does not contain second-order time derivatives.

A Hamiltonian formalism may be built up in terms of u and g_{kl} and their conjugate variables, say, w and p^{kl}. In principle, any reasonable set of Cauchy conditions on a hypersurface $t = 0$ is acceptable, since there are no constraints.

From (2.2), and because of the contracted Bianchi identities, we have, as in general relativity,

$$D_\nu G^\nu_\mu = 0 \tag{2.13}$$

and if we take account of (2.11) we may reduce (2.13) to the following set of equations for $\mathscr{H}_k \equiv g_{kl}\mathscr{H}^l \equiv (-g)^{1/2} G^0_k$:

$$(u^4 G_{00})^\bullet + u^2\, \partial_k(\Phi^2 \mathscr{H}^k) = 0 \tag{2.14}$$

$$\dot{\mathscr{H}}_k - u^2\, \partial_k G_{00} = 0 \tag{2.15}$$

We shall use these equations below.

3. *Cosmological Models*

For certain homogeneous universes like Bianchi type IX, for isotropic and homogeneous worlds, as well as in other cases of interest, $\mathscr{H}_k = 0$ identically for symmetry reasons. When this is the case, we see from (2.14) and (2.15) that G_{00} depends on t only and that

$$G_{00}(t) = F(x^k) u^{-4} \tag{3.1}$$

in which F is an arbitrary function of x^k. We shall obtain the same result if we note that our equations (2.9) and (2.10) may now be written as follows:

$$G^\nu_\mu = \Theta^\nu_\mu \tag{3.2}$$

in which

$$\Theta^\nu_\mu \equiv \operatorname{diag}(v, -v, -v, -v) \tag{3.3}$$

with

$$v \equiv a\ddot{u}/u\Phi^2 \tag{3.4}$$

Then, because of (2.13), (3.2) gives

$$D_\nu \Theta^\nu_\mu = 0 \tag{3.5}$$

and this leads us to the same result (3.1) provided we take account of (2.12). It turns out that (2.12) and (3.1) are integrable and the solution may be written as follows:

$$G^0_0 = F/Q^2 \tag{3.6}$$

$$\Phi = Q/(t^2 + 1/\epsilon a)(\epsilon F)^{1/2} \qquad \epsilon \equiv \operatorname{sign} F \tag{3.7}$$

Φ is a regular function for any value of t if $\epsilon a > 0$. In writing (3.7) we made use of the freedom of choosing t as defined in (2.3); t is now fixed up to its sign.

Let us consider in particular a Robertson–Walker metric which in co-moving coordinates may be written as:

$$ds^2 = \Phi^2(t)\,dt^2 - \psi^2(dx^2 + dy^2 + dz^2) \tag{3.8}$$

with

$$\psi \equiv S(t)/(1 + \tfrac{1}{4}kr^2); \qquad k = 0, \qquad \pm 1; \qquad r^2 \equiv x^2 + y^2 + z^2 \tag{3.9}$$

Equations (2.11) and (2.12) now take the following form: (2.11) reduces to one non-trivial equation for the one significant function S whose integral is just (3.6), and the solution of (2.12) is (3.7). However since Φ depends only on t it follows that F is now defined up to a constant of integration $A > 0$ and

$$F = \epsilon A^2/(1 + \tfrac{1}{4}kr^2)^6 \tag{3.10}$$

With (3.10), (3.6) and (3.7) we now have the following structure:

$$G_0^0 = \epsilon A^2/S^6 \tag{3.11}$$

$$\Phi = S^3/A(t^2 + 1/\epsilon a) \tag{3.12}$$

Equation (3.11) is a modified Friedman equation that may be written in a more familiar form by introducing a new variable

$$\tau \equiv \int_0^t \Phi(\hat{t})\,d\hat{t} \tag{3.13}$$

With a prime indicating a derivative relative to τ, (3.11) now appears as follows:

$$3(S'^2 + k)S^{-2} - T_0^0 = \epsilon A^2 S^{-6} \tag{3.14}$$

An S^{-6} modification appears in a variety of modified Friedman equations, for instance in Hoyle & Narlikar (1964), in Rosen (1969) and in Trautman (1973). In all these cases as well as in (3.14) the origin of the S^{-6} term is the same: any modification like Θ_μ^ν, of the energy tensor T_μ^ν, has necessarily the same form as T_μ^ν itself, in the special universe (3.8), namely

$$T_\mu^\nu \equiv \mathrm{diag}(\sigma, -p, -p, -p) \tag{3.15}$$

with at most two significant different functions; if these two functions are equal, and if (3.5) is satisfied, then Θ_0^0, like in the second member of (3.11), is an S^{-6} term.

Equation (3.14) is thus well known. For $k = 0$ it gives a contracting–dilatating world with one real bounce and, when $k = +1$, the solution is a perpetually oscillating closed universe that does not go through a singular state, *provided in both cases that* $\epsilon = -1$. In view of (3.12), one obtains a regular metric for any value of t if the coupling constant a is negative: $a < 0$.

Before trying to find some way of evaluating a, or at least put an upper bound on it, it may be of interest to note that (2.12) may also be written in the following form:

$$-(\ddot{\Phi}/\Phi^3) + 3(\dot{\psi}^2/2\Phi^2\psi^2) + 3(\dot{\Phi}^2/2\psi^4)$$
$$+ \sum_k [(\psi\, \partial_{kk}\Phi + \partial_k\Phi\, \partial_k\psi)/\Phi\psi^3] - \tfrac{1}{2}(T_0^0 - T_k^k) = 2(1 + 1/a)G_0^0 \tag{3.16}$$

For $a = -1$, this equation is the same as the one for Φ used by Goldman & Rosen (1972) and is one of the equations of Rosen's preferred frame theory of gravitation (Rosen, 1971a, b).

4. *Post-Newtonian Approximation and the New Coupling Constant*

Consider now the case of a perfect fluid with an energy tensor of the form of (3.15). In non-comoving coordinates,

$$T^{\mu\nu} = (\sigma + p)u^\mu u^\nu - g^{\mu\nu}p \tag{4.1}$$

u^λ is the velocity field of matter. Consider the so-called Post Newtonian (PN) approximation (Chandrasekar, 1965). We shall see below that our equations (2.11) and (2.12) lead to the same approximate PN metric as general relativity. This will imply at least two things:

(i) Experimental accuracy does not go beyond the PN approximation at present. Since the PN approximation of general relativity agrees with all present experimental evidence concerning gravitation (Nordtvedt & Will, 1972) it follows that our preferred frame theory also agrees with all present observations. This rather unusual property of a preferred frame theory is worth noting.

(ii) In addition, it implies that no fine upper-bound can be put for the moment on the coupling constant a on the basis of solar system experiments. What is needed is a measurement whose sensitivity is of the order of a post-PN term. Such a measurement may be achieved in another decade or so from now if progress in experimental techniques continues at the present rate.

Let us now very briefly show that the PN-approximate solution of (2.2), (2.11) and (2.12) is the same as that of general relativity. The object of a PN formalism is to calculate the dynamical equations (2.2) up to terms of order c^{-6} or briefly up to† 0(6). To do this we need to obtain g_{00} up to 0(6), g_{0k} up to 0(5) and g_{kl} up to 0(4). Regarding $T^{\mu\nu}$, it is useful to note the order in c^{-1} of some of its terms since in our units this is not apparent at first sight. As usual σ may be split into $\rho + \epsilon$, where ρ is the rest mass energy density and ϵ the proper internal energy density. So ρ is 0(2), ϵ and p are 0(4) and $u^k = v^k + 0(2)$, v^k being 0(1).

† Up to 0(6) means including terms of order 0(4) but neglecting terms of order 0 ($\geqslant$ 6).

We shall use the PPN formalism of Will & Nordtvedt (1972) (to whom we shall refer for details). We shall however transform the PPN metric to coordinates in which $g_{0k} = 0$. This will considerably simplify the calculations with (2.11) and (2.12). It may be shown after a little computation that in coordinates in which $g_{0k} = 0$, the PPN form of g_{00} and g_{kl} are as follows for any metric theory:

$$g_{00} = 1 + 2U + g_{00}(4) + 0(6) \tag{4.2}$$

$$g_{kl} = -\delta_{kl} + 2\gamma U\delta_{kl} - (7\Delta_1 + \Delta_2) \int^t \partial_{(k} V_{l)}\, dt' + 0(4) \tag{4.3}$$

here, γ, Δ_1, Δ_2 are parameters, $g_{00}(4)$ is some elaborate expression of order 0(4) whose explicit structure we shall not need, while

$$U(x^k, t) \equiv -\frac{1}{8\pi} \int \frac{\rho(x'^k, t)}{R'}\, d^3x', \qquad R'^k \equiv x^k - x'^k \tag{4.4}$$

and

$$V_k(x^k, t) \equiv +\frac{1}{8\pi} \int \frac{\rho(x'^l, t) v^k(x'^m, t)}{R'}\, d^3x' \tag{4.5}$$

or, equivalently,

$$\sum_k \partial_{kk} U \equiv \Delta U = \tfrac{1}{2}\rho, \qquad \Delta V_k = -\tfrac{1}{2}\rho v^k \tag{4.6}$$

One has to find γ, $\lambda \equiv 7\Delta_1 + \Delta_2$ and $g_{00}(4)$; for this we shall insert $g_{\mu\nu}$ as given by (4.2) and (4.3) into equations (2.11) and (2.12), which we shall decompose according to their order in c^{-1}; symbolically we shall write (2.11) and (2.12) as follows:

$$W_{kl}(2) + W_{kl}(4) = 0(6) \tag{4.7}$$

$$G_{00}(2) + G_{00}(4) = a\ddot{u}/u + 0(6) \tag{4.8}$$

In general relativity $a = 0$, in addition one has three more equations:

$$G_{0k} \equiv G_{0k}(3) + 0(5) = 0 \tag{4.9}$$

We consider now the following steps in our proof:

(i). Since—see (4.9)—G_{0k} is 0(3) and since $\ddot{u}$ is obviously at least 0(4), the equations (4.7) and (4.8) to order two, namely

$$W_{kl}(2) = G_{00}(2) = 0 \tag{4.10}$$

are the same as in general relativity; from which follows, as in general relativity, $\gamma = 1$ but, as results from explicit calculation, λ is not determined.

(ii) Consider next equations (2.14) and (2.15), that is

$$(u^4 G_{00})^{\cdot} + u^2\, \partial_k(\mathscr{H}^k \Phi^2) = 0 \tag{4.11}$$

$$\dot{\mathscr{H}}_k - u^2\, \partial_k G_{00} = 0 \tag{4.12}$$

With (4.10), G_{00} in (4.11) and (4.12) is $0(4)$. Thus, to the lowest order in c^{-1}, we may write, if we use (4.8), these equations as follows:

$$\sum_k \partial_k G_{0k} = 0(5) \tag{4.13}$$

$$(G_{0k} - a\,\partial_k \dot{u})^{\cdot} = 0(6) \tag{4.14}$$

in which, to order $0(3)$, G_{0k} may be shown to have the following form

$$G_{0k} \equiv \tfrac{1}{4}(8-\lambda) \sum_l (\partial_{lk} V_l - \partial_{ll} V_k) + 0(5) \tag{4.15}$$

while

$$\dot{u} = -\tfrac{1}{4}(8-\lambda)\dot{U} = -\tfrac{1}{4}(8-\lambda) \sum_l \partial_l V_l \tag{4.16}$$

Note in both cases the factor $(8-\lambda)$. With (4.15) we see that (4.13) is identically satisfied and with (4.15) and (4.16) we see that (4.14) to order $0(4)$ can be satisfied only if $\lambda = 8$, which is the same value as in general relativity. This implies that $\mathscr{H}_k$ is $0(5)$ and $\ddot{u}$ is $0(6)$.

(iii) Consider finally the equations for $g_{00}(4)$. Since $\ddot{u}$ is $0(6)$, (4.7) and (4.8) are up to $0(6)$, the same as in general relativity. It follows from this that $g_{00}(4)$ is also the same as in general relativity and thus the PPN formalism of our equations is the same as that of general relativity.

5. *Conclusions*

We have described a preferred frame metric theory of gravitation which agrees with all present observational evidence. It admits cosmological solutions of an oscillating type with real 'bounces'. This has been obtained at the expense of one new coupling parameter which is at present not measurable but should become so in the more or less near future. In certain limited conditions, the field equations reduce to those of general relativity.

We should finally mention that another preferred frame theory has been developed recently by Wei-Tou-Ni (1973). It is notably different from the present one: it contains adjustable functions and the PPN approximation *may* agree with that of general relativity though the equations will never reduce to Einstein equations.

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